Clustering-Based Robot Navigation and Control

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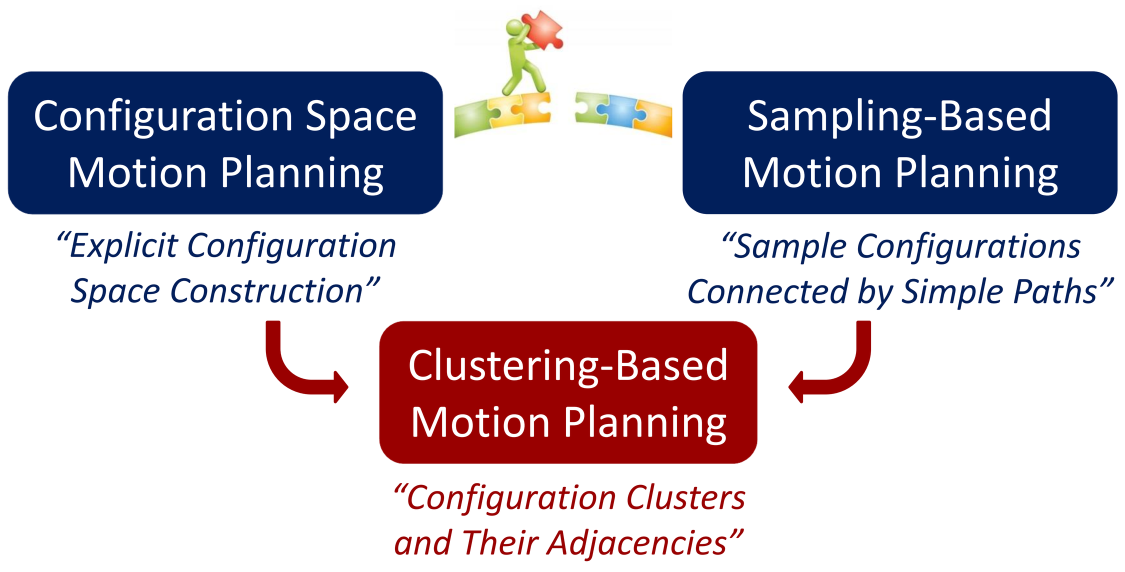
Introduction

In robotics, it is essential to model and understand the topologies of configuration spaces in order to design provably correct motion planners. The common practice in motion planning for **modelling configuration spaces requires either a global, explicit representation of a configuration space** in terms of standard geometric and topological models, **or an asymptotically dense collection of sample configurations** connected by simple paths. As a new alternative approach, **we propose the use of *clustering* for closing the gap** between these two complementary approaches to combine their strengths.

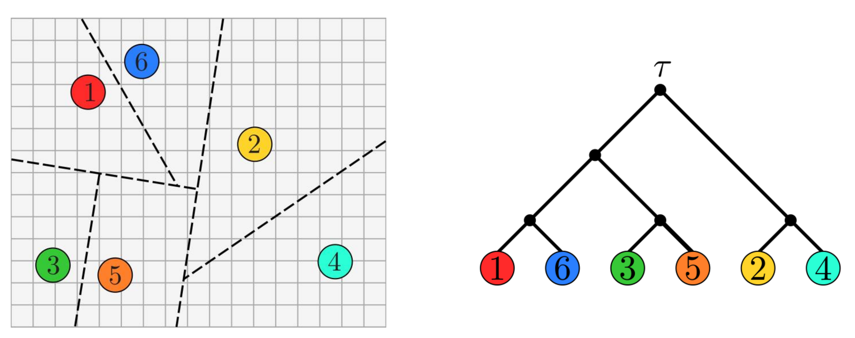


Clustering enables us to **discover hidden intrinsic structures** in generally complex shaped and high-dimensional configuration spaces. We argue that the intrinsic local structures in configuration spaces that are identified by clustering can be exploited to **design computationally efficient, provably correct motion planners.**

What Does Clustering Offer?

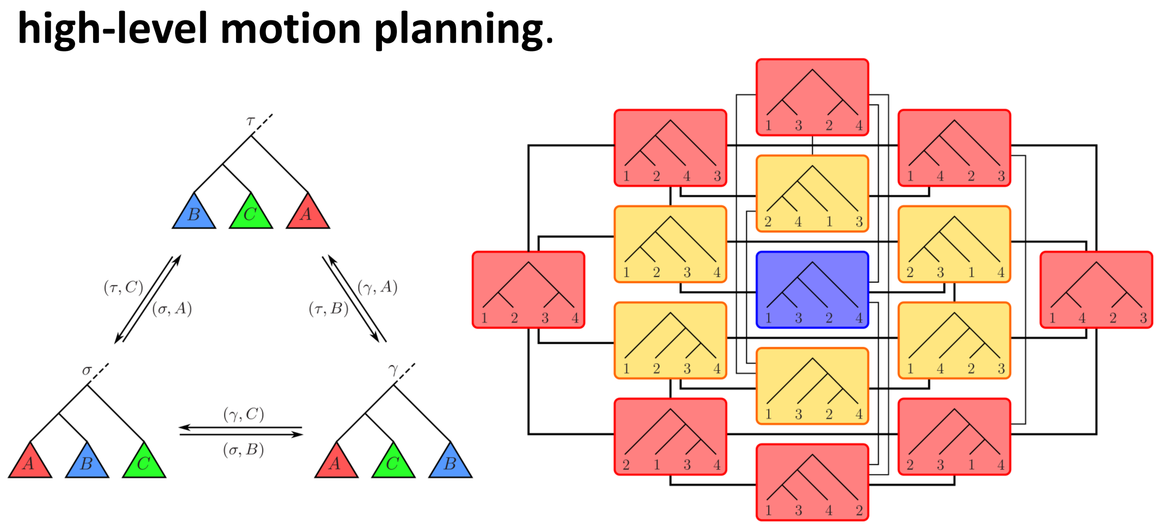
Traditionally an unsupervised learning method, *clustering* offers **automated tools to discover coherent groups in configuration spaces** to model their unknown global organizational structure (e.g., hierarchical clustering), and **to determine collision-free local neighborhoods** of robot configurations (e.g., partitional clustering) [1].

On a more conceptual level, clustering can be viewed as **a symbolic abstraction relating the continuous space of configurations to the (combinatorial) space of clustering models** (e.g., cluster hierarchies and finite set partitions) .



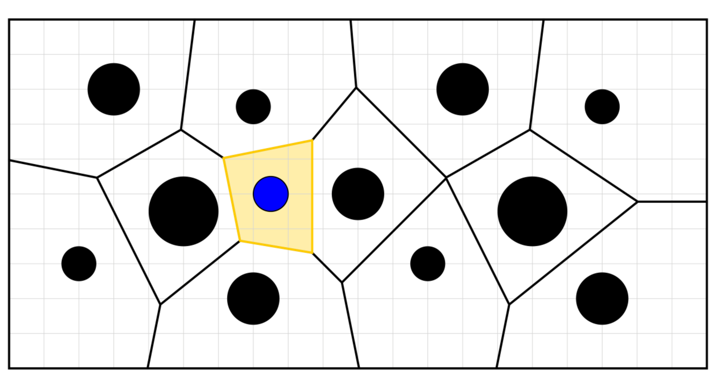
A multirobot configuration and its cluster hierarchy

In consequence, explicit relations between clustering models can be exploited to **reduce the complexity of high-level motion planning**.



The nearest neighbor interchange (NNI) moves between cluster hierarchies and the NNI graph

Another characteristic use of clustering is for locality identification. One can utilize clustering to identify a collision-free neighborhood of a robot that also captures **the local geometric structure of the configuration space around the robot's instantaneous position.**



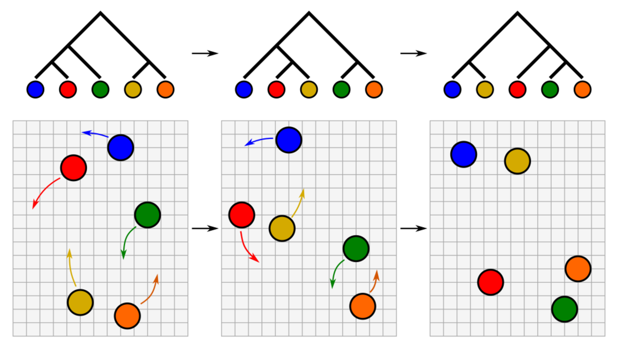
Robot-centric Voronoi diagrams

Coordinated Robot Navigation via Hierarchical Clustering

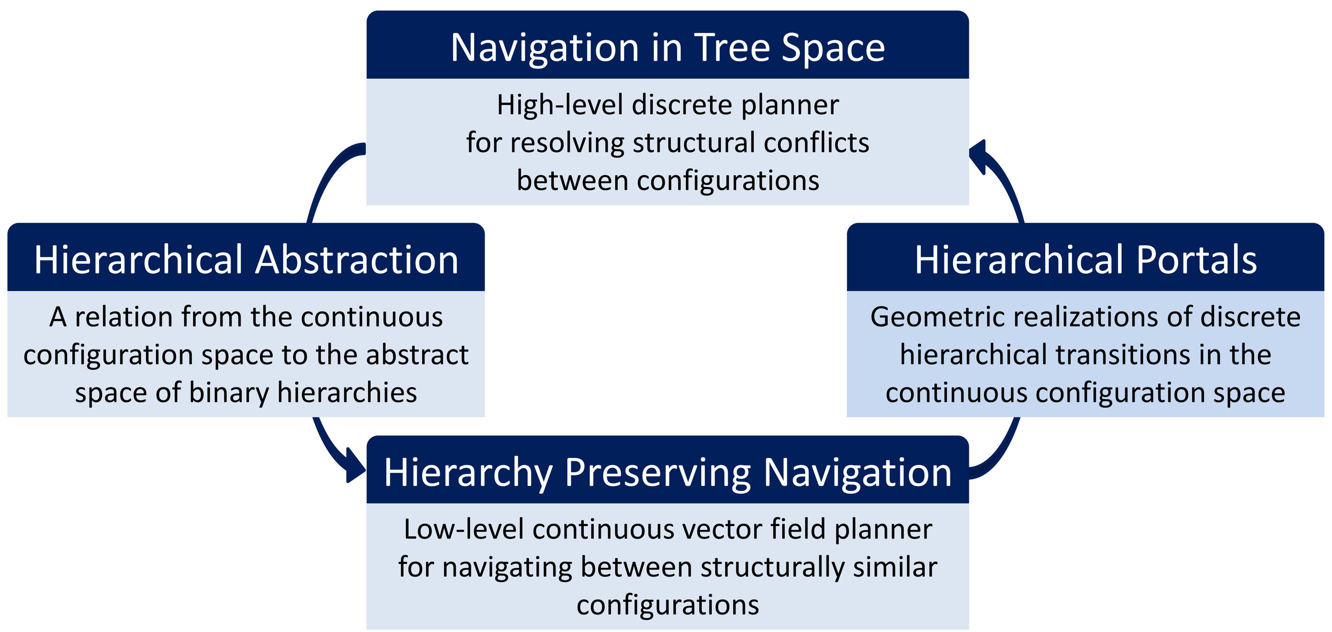
Hierarchical clustering offers **a natural abstraction** for ensemble task encoding and control of multirobot systems in terms of **precise yet flexible organizational specifications at selectively multiple resolutions**. This abstraction intrinsically suggests a **two-level navigation strategy** for coordinated motion design [6]:

1) **At the low-level, perform finer adjustments** on configurations by using hierarchy preserving vector fields [3];
2) **At the high-level, resolve structural conflicts** between configurations by using a discrete transition policy in tree space [5].

The connection between these two levels is established by an optimal selection of a portal configuration supporting two adjacent hierarchies [4].



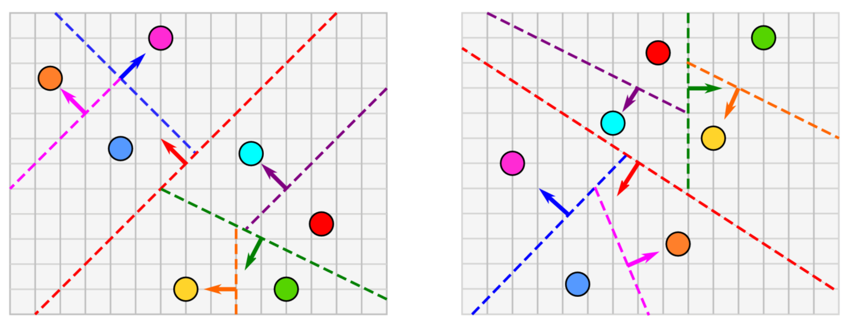
Moving from one spatial distribution to another is generally carried through rearrangements of robot groups (clusters) at different resolution corresponding to transitions between different cluster structures (hierarchies).



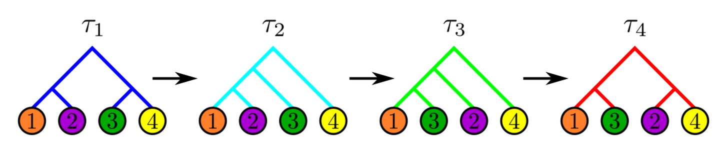
Generic components of hierarchical navigation framework

Topological Shape of a Hierarchical Stratum

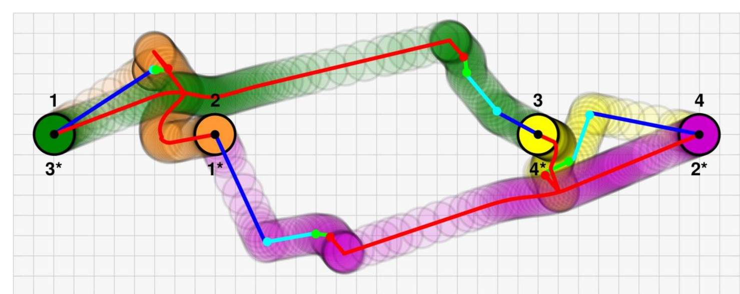
Theorem: The homotopy model of configurations sharing the same cluster hierarchy is a generalized torus, $\left(\mathbb{S}^{d-1}\right)^{n-1}$.



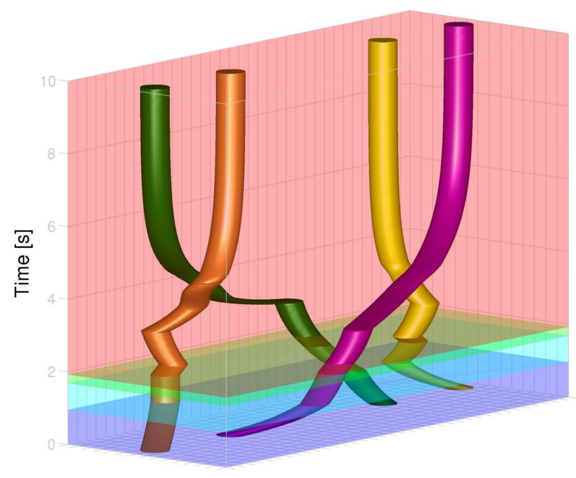
Navigation in a hierarchical stratum is accomplished by aligning separating hyperplanes of configurations



Sequence of trees associated with deployed local controllers



Navigation paths of disk-shaped robots

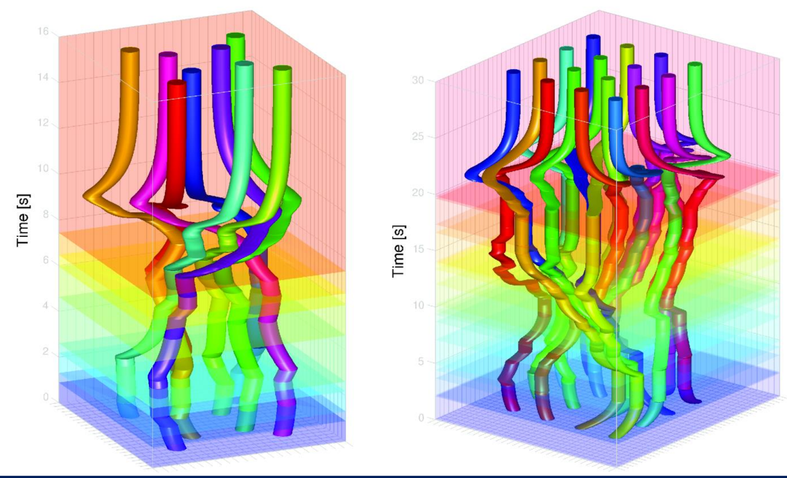


Space-time curve of robots

Computational Properties

- Hierarchy preserving navigation is computable in $O(n^2)$ time.
- Navigation in tree space requires at most $O(n^2)$ steps, each step costing $O(n)$ computations.

	Number of Robots, n				
Local Policy	4	8	12	16	n
Total	15	$> 10^5$	$> 10^{10}$	$> 6 \times 10^{15}$	$(2n-3)!!$
Max. Deployed	3	21	55	105	$0.5(n-1)(n-2)$

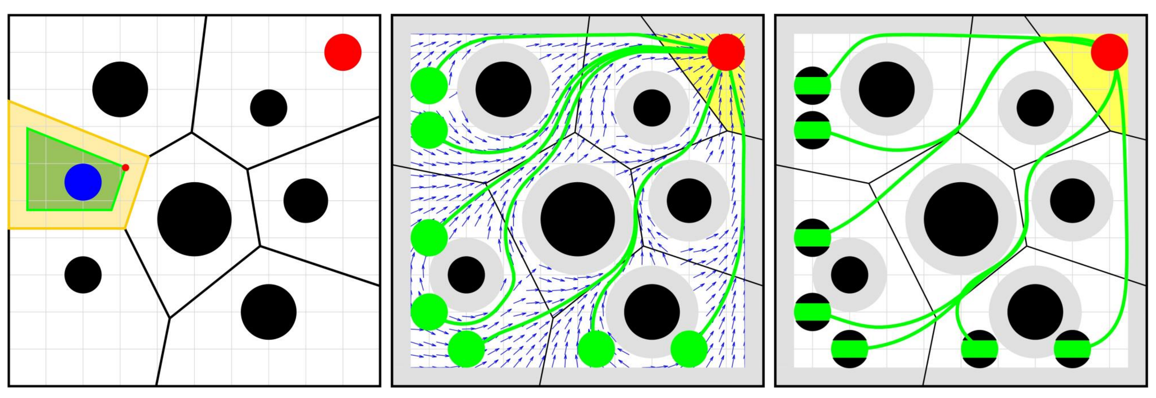


For the case of groups of 8 and 16 disks, our planner only deploys 9 and 19 local controllers, respectively.

Encoding Collisions via Robot-Centric Voronoi Diagrams

We introduce a new, robot-centric application of Voronoi diagrams to **encode robot collisions exactly** by exploiting the local structure of configuration spaces around a robot configuration [7,8]. This also enables us to determine a **safe convex neighborhood** of a robot configuration.

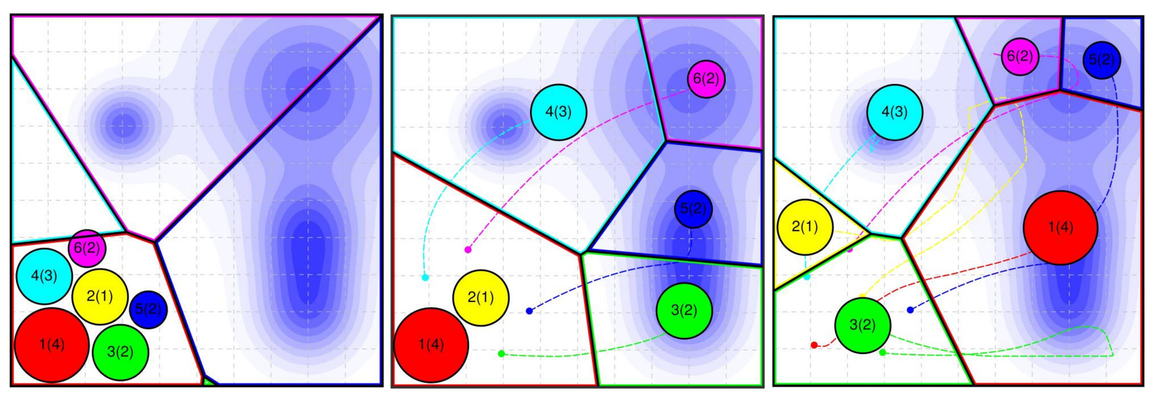
We show that the continuous feedback **motion toward the metric projection of a desired goal onto the robot's convex Voronoi cell** steers almost all robot configurations to the goal in environments cluttered with spherical obstacles, while avoiding collisions along the way [7]. We observe that the **robot balances its distance to all proximal obstacles while navigating toward the goal**.



Exact robot navigation using robot-centric Voronoi diagrams

In distributed mobile sensing applications, Voronoi diagrams are often utilized for solving sensory task assignment and for modelling group heterogeneity in actuation, sensing, computation and energy sources [2]. In addition to these usages, we tailor Voronoi diagrams to **encode collisions in a heterogeneous group of disk-shapes robots**.

Based on standard coverage control of point robots [2], we propose **a constrained coverage control law** for heterogeneous disk-shaped robots that **solves the combined sensory coverage and collision avoidance problem** [8]. We further introduce a **congestion management heuristic** for unassigned robots to hasten the assigned robots' progress.



Safe coverage control of heterogeneous disk-shaped robots

Conclusion

We introduce **the use of clustering for modelling configuration spaces and for design of provably correct motion planners**. This new philosophy for modelling configuration spaces, still in its infancy, yields promising results for closing the gap between standard configuration space and sampling-based motion planning approaches. We demonstrate some potential applications of clustering to the problem of feedback motion planning and control. We believe that these nontrivial applications of clustering to robot motion design only scratch the surface of its long-term potential.

References

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